

Lattice Supersymmetry

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A method is proposed for latticizing a class of supersymmetric gauge theories, including $N=4$ super Yang-Mills theory. The technique is inspired by recent work on “deconstruction”. Part of the target theory’s supersymmetry is realized exactly on the lattice, reducing or eliminating the need for fine tuning. (Talk based on the paper *Supersymmetry on a Spatial Lattice*, hep-lat/0206019, by D.B.K., Emmanuel Katz and Mithat Unsal)

1. Exact supersymmetry on the lattice

Supersymmetric gauge theories are expected to exhibit various fascinating phenomena, including electromagnetic duality, nontrivial conformal fixed points, monopole condensation, and relations to gravity and string theory through the AdS/CFT correspondence. It is desirable to study these theories nonperturbatively, and the lattice is the obvious tool. There has been much work done on lattice supersymmetry, but the prospects for practical success presently seem limited.

The origin of the problem is that supersymmetry is part of the super-Poincaré group, which is explicitly broken by the lattice. Ordinary Poincaré invariance is also broken in lattice QCD, for example, but due to the hypercubic symmetry, operators which violate Poincaré symmetry are all irrelevant. In a supersymmetric theory, however, the lattice point group is never sufficient to forbid all relevant or marginal supersymmetry violating operators. The most benign four dimensional supersymmetric gauge theory is $N = 1$ super Yang-Mills (SYM), consisting of gauge bosons and gauginos. In this theory, the only relevant SUSY violating operator is the gaugino mass. One can therefore tune to the massless point (which has an enhanced Z_{2N_c} symmetry), or use chirally improved fermions such as domain wall or overlap fermions to eliminate the fine-tuning

(see [?] and references therein).

However, most supersymmetric theories contain scalar bosons, such as $N = 2$ or $N = 4$ SYM, or $N = 1$ theories with matter fields. When scalars are present there are typically a plethora of possible SUSY violating relevant operators to fine tune away, a practically impossible task. The only symmetry that can prevent these unwanted operators is supersymmetry itself. It is therefore natural to ask whether one can construct lattices which realize exactly at least *some* of the target theory’s supersymmetry, with the hope of ameliorating the fine tuning problem. In this talk I describe a recent attempt along this line [?] (for other recent ideas about lattice supersymmetry, see [?, ?]). Presented here are spatial lattices in Minkowski time; construction of the more useful Euclidean spacetime lattices is underway.

2. Lattices from orbifolding

Motivated by recent work on deconstruction of supersymmetric theories [?, ?] (see also [?]), we construct our spatial lattices by “orbifolding”. We start with a non-latticized “mother theory” which exists in $0 + 1$ dimensions, which is a $U(kN^d)$ gauge theory possessing the amount of supersymmetry desired of the target theory. It will also possess a global symmetry (called an R -symmetry) which does not commute with supersymmetry. To create the lattice we now iden-

tify a Z_N^d subgroup embedded within both the gauge and global symmetry groups of the mother theory; the embedding is discussed in ref. [?]. We now project out all field components in the mother theory which are not singlets under this Z_N^d symmetry. After projection, adjoint fields of the mother theory, written as $kN^d \times kN^d$ matrices, are zero everywhere except for N^d $k \times k$ blocks on or near the diagonal. These can be interpreted as fields with near-neighbor interactions living on a d -dimensional spatial lattice with N^d sites. The orbifold projection breaks the gauge symmetry down to $U(k)^{N^d}$, appropriate for a $U(k)$ gauge theory in the continuum. The orbifold projection also breaks some of the mother theory's supersymmetries, half for each Z_N factor. The various components of the mother theory's supermultiplets get spread about the lattice within approximately one lattice spacing of each other, so that the breaking of the mother theory's supersymmetry is rather benign, and is restored in the continuum limit.

The resulting lattices are quite peculiar and wonderful: there are one-component fermions scattered over links and sites; noncompact link variables that become gauge fields; bosonic variables on links transforming non-trivially under the lattice point symmetry which become spin-zero particles in the continuum. There is no fermion doubling problem (the exact residual supersymmetry of the lattice ensures that) and yet one can realize without fine tuning continuum target theories which exhibit chiral symmetries.

3. A 1 + 1 dimensional example

As an example of the type of lattice that results, consider our simplest case: a spatial lattice whose target theory is (2, 2) SYM in 1 + 1 dimensions. (The “(2, 2)” designation means that there are four real chiral supercharges, two left- and two right-handed) The target theory consists of a $U(k)$ gauge field with coupling g_2 , a Dirac fermion Ψ , and a complex scalar S (all $U(k)$ adjoints) with the Lagrangian

$$\mathcal{L} = \frac{1}{g_2^2} \text{Tr} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\Psi} i \not{D} \Psi - (D_\mu S)^\dagger (D^\mu S) \right.$$

$$\left. + \sqrt{2} (\bar{\Psi}_L [S, \Psi_R] + h.c.) - \frac{1}{2} [S^\dagger, S]^2 \right) \quad (1)$$

To obtain the lattice theory we start with a 0 + 1 dimensional mother theory with a $U(kN)$ gauge symmetry and four exact supercharges. This mother theory is easily constructed by taking the familiar $N = 1$ SYM theory in 3 + 1 dimensions, and dimensionally reducing it to 0 + 1 dimensions (that just means: take the gauge and gaugino fields to be independent of the x, y, z coordinates). This mother theory then has four real bosonic fields and four real fermionic fields. It possesses a global R symmetry which is $U(1) \times SO(3)$; the $U(1)$ is the same $U(1)$ R -symmetry found in the 3 + 1 dimensional $N = 1$ SYM theory, while the $SO(3)$ is just the rotational group that is inherited as an internal symmetry after the dimensional reduction.

We now orbifold the theory, modding out by a Z_N symmetry which is embedded in this $U(kN) \times U(1) \times SO(3)$ symmetry of the mother theory, as described in ref. [?]. The effect of the orbifolding is to create an N -site, one dimensional periodic lattice. At each site lives a gauge field v_0 , a real scalar σ and a complex one-component fermion λ . On each link lives a complex scalar field ϕ and a complex one-component fermion ψ . Each field is a $k \times k$ matrix, and there is an independent $U(k)$ gauge symmetry associated with each site. Orbifolding breaks up the multiplet of the mother theory, and reduces the exact supersymmetry from four supercharges to two. Under the residual supersymmetry, the site fields form a real (“vector”) supermultiplet, while the link fields form a “chiral” supermultiplet. In component form, the lattice Lagrangian takes the form:

$$\begin{aligned} L = \frac{1}{g^2} \sum_n \text{Tr} \left[\frac{1}{2} (D_0 \sigma_n)^2 + \bar{\lambda}_n i D_0 \lambda_n + |D_0 \phi_n|^2 \right. \\ + \bar{\psi}_n i D_0 \psi_n - \bar{\lambda}_n [\sigma_n, \lambda_n] + \bar{\psi}_n (\sigma_n \psi_n - \psi_n \sigma_{n+1}) \\ - |\sigma_n \phi_n - \phi_n \sigma_{n+1}|^2 - \frac{1}{2} (\phi_n \bar{\phi}_n - \bar{\phi}_{n+1} \phi_{n+1})^2 \\ \left. - \sqrt{2} (i \bar{\phi}_n (\lambda_n \psi_n + \psi_n \lambda_{n+1}) + h.c.) \right], \quad (2) \end{aligned}$$

where

$$D_0 \phi_n = \partial_0 \phi_n + i v_{0,n} \phi_n - i \phi_n v_{0,n+1} \quad (3)$$

and similarly for $D_0 \psi_n$. However, this form hides

the supersymmetry of the lattice. If instead one writes the Lagrangian in terms of the appropriate superfields, one finds the simple form

$$L = \frac{1}{g^2} \sum_{n=1}^N \text{Tr} \left[\frac{1}{2} \bar{\Phi}_n i \mathcal{D}_0^- \Phi_n + \frac{1}{8} \bar{\Upsilon}_n \Upsilon_n \right]_{\tilde{\theta}\theta} . \quad (4)$$

where Υ_n is the Grassman chiral multiplet containing the gauge kinetic terms at site n .

This Lagrangian has a classical moduli space (that is: noncompact flat directions for boson field vevs). We now expand about the vev $\phi_n = \frac{1}{\sqrt{2}a} \times \mathbf{1}_k$, where $\mathbf{1}_k$ is the $k \times k$ unit matrix and a will be the scale defining the lattice spacing. If we define the target theory's coupling g_2 in terms of the lattice coupling g as $g_2^2 \equiv ag^2$ and lattice size $L = Na$, then the continuum limit is $a \rightarrow 0$ and $N \rightarrow \infty$ for fixed $L^2 g_2^2$. At tree level we recover the target theory of Eq. (1), with the identification $A_0 = v_0$ and $A_1 = \text{Im}\phi$ (where $A_{0,1}$ are the gauge fields of the target theory) and

$$S = \frac{\sigma + i \text{Re}\phi}{\sqrt{2}} , \quad \Psi = \left(\frac{\psi}{\lambda} \right) . \quad (5)$$

Dimensional analysis reveals that the only dangerous relevant or marginal operators are scalar tadpoles and mass terms; however the two exact supersymmetries preclude generating local counterterms for these operators. One must, however, control the noncompact flat directions, or else the path integral is not defined. This can be done by fixing initial and final data on all bosonic zero-modes, and by restricting correlation measurements to time separation no longer than the spatial size of the lattice.

4. Other lattices

We have constructed spatial lattices corresponding to the continuum super Yang-Mills theories with four real supercharges in $1+1$ dimensions (described above); eight real supercharges in $1+1$ and $2+1$ dimensions; and sixteen real supercharges in $1+1$, $2+1$ and $3+1$ dimensions ($N = 4$ SYM). These lattices have unusual structure, such as hexagonal and body-centered cubic for the sixteen supercharge lattices. In each case but the last we can use the

lattice symmetries to argue that in perturbation theory there are no allowed counterterms for relevant or marginal operators that would violate the symmetries of the target theory; since these are all super-renormalizable theories, this argument should suffice for proving that no fine tuning is needed in the continuum limit. For the four dimensional case, we have no symmetry argument to forbid dimension four operators that would violate the desired $N = 4$ supersymmetry. However, we believe that an anisotropic lattice, where two spatial dimensions are characterized by a shorter lattice spacing than the third, can have an $N = 4$ continuum limit without fine tuning, where would-be log divergences are suppressed by a ratio of lattice spacings, which can be taken to zero.

Euclidean spacetime lattices may be constructed by orbifolding certain supersymmetric matrix models. It remains to be seen how much fine tuning, if any, is needed for these lattices. An encouraging result, however, is that the fermion “determinants” for these theories are real and positive, allowing for Monte Carlo simulation without a sign problem.